## Cambridge International Examinations

Cambridge International General Certificate of Secondary Education

## CANDIDATE NAME

CENTRE NUMBER


CANDIDATE NUMBER

## CAMBRIDGE INTERNATIONAL MATHEMATICS

0607/61
Paper 6 (Extended)
May/June 2016
1 hour 30 minutes
Candidates answer on the Question Paper.
Additional Materials: Graphics Calculator

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO NOT WRITE IN ANY BARCODES.

Answer both parts $\mathbf{A}$ and $\mathbf{B}$.
You must show all the relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40 .

## Answer both parts A and B.

## A INVESTIGATION

## MOVING TRIANGLES (20 marks)

You are advised to spend no more than 45 minutes on this part.


This investigation is about finding the connection between $A P$ and $B Q$ as $P$ and $Q$ move.
All triangles are right-angled and $R S$ is one unit.
In questions $\mathbf{1 , 2}$ and $\mathbf{3}, P$ is at $A$.

1


Triangle $P B Q$ is an enlargement of triangle $P S R$, with $P$ as the centre of the enlargement.
(a) Write down the scale factor of the enlargement in the diagram.
(b) Complete the table for enlarging triangle $P S R$.

| Scale factor | Length of $P S$ | Length of $P B$ |
| :---: | :---: | :---: |
| 3 | 4 |  |
| 7 | 6 | 30 |
|  |  | 14 |

(c) Use one word to complete this statement.
$P S R$ and $P B Q$ are $\qquad$ triangles because one is an enlargement of the other.

(a) Show that $x=10$ when $P B=20$.
(b) Find the value of $x$ when $P B=16$.
(c) Find an expression for $x$ when $P B=y$.

3


NOT TO
SCALE

For this triangle, find an expression for $x$ when $P B=y$.

4 These diagrams show $P$ starting at $A$ and then moving towards $B$.
(a)


NOT TO
SCALE

In this diagram $P$ is at $A$.
Find the value of $y$.
(b)


NOT TO
SCALE

In this diagram, $P$ and $S$ have moved towards $B$.
$P S$ is one unit less than in part (a).
Find the value of $z$.
(c) Using your answers to part (a) and part (b), work out the value of $A P$.


These diagrams show $P$ starting at $A$ and moving towards $B$.
In the second diagram, $P S$ has decreased by one unit.
Using the method of question $\mathbf{4}$, show that $A P=B Q$.


These diagrams show $P$ starting at $A$ and moving towards $B$. In the second diagram, $P S$ has decreased by one unit.

Show that $A P=B Q$.

7 In this question, $R S$ is no longer one unit.
$P$ starts at $A$ and moves towards $B$.
In the second diagram, $P S$ has decreased by one unit.

(a) When $R S=2$, find an expression for $A P$ in terms of $x$.
(b) When $R S=m$, find an expression for $A P$ in terms of $x$ and $m$.

## B MODELLING

## MUSICAL NOTES (20 Marks)

You are advised to spend no more than 45 minutes on this part.
This task is about the connection between musical notes.
A musical note is made by a sound wave which is modelled by a sine function.
Here is the sine wave for the note $\mathrm{A}_{1}$, where the time, $t$, is measured in seconds.


Each note has a different frequency, which is measured in Hertz (Hz).
The frequency of the note $\mathrm{A}_{1}$ is 55 Hz because the sine wave repeats 55 times per second.

1 The frequency of the note $\mathrm{A}_{2}$ is two times the frequency of the note $\mathrm{A}_{1}$. On the grid below, sketch the sine wave for the note $\mathrm{A}_{2}$ for $0 \leqslant t \leqslant \frac{2}{55}$.


2 The 12 notes in a musical scale are A, A\#, B, C, C\#, D, D\#, E, F, F\#, G, G\#. The notes on a piano repeat this scale.
Notes in the same scale have the same subscript. (For example, $\mathrm{A}_{2}, \mathrm{CH}_{2}$ and $\mathrm{F}_{2}$ are all in the same scale.)
The frequency, fHz , of each note on a piano is modelled by the function

$$
\mathrm{f}(n)=27.5 \times 2^{\frac{n}{12}} \quad \text { where } n \text { is an integer from } 0 \text { to } 87 .
$$

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 87 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Note | $\mathrm{A}_{0}$ | $\mathrm{~A}_{0}$ | $\mathrm{~B}_{0}$ | $\mathrm{C}_{0}$ | $\mathrm{C} \#_{0}$ | $\mathrm{D}_{0}$ | $\mathrm{D} \#_{0}$ | $\mathrm{E}_{0}$ | $\mathrm{~F}_{0}$ | $\mathrm{~F} \#_{0}$ | $\mathrm{G}_{0}$ | $\mathrm{G} \#_{0}$ | $\mathrm{~A}_{1}$ | $\mathrm{~A} \#_{1}$ | $\mathrm{~B}_{1}$ |  |

(a) When $n=0$ the frequency of the note is 27.5 . This is the note $\mathrm{A}_{0}$.
(i) Work out the frequency of the note when $n=3$.
(ii) Write down the note when $n=15$.
(iii) Work out the frequency of the note $\mathrm{E}_{0}$.
(b) Write down all the values of $n$ that give the note A on this piano.
$\qquad$
(c) Which note has the highest frequency on this piano?

Calculate this frequency.
$\qquad$
$\qquad$
$3 k$ times the frequency of a note gives the frequency of the next note. This means that $k \mathrm{f}(n)=\mathrm{f}(n+1)$.

Find the exact value of $k$.

4 (a) On the axes below, sketch the graph of $g(x)=27.5 \times 2^{\frac{x}{12}}$ for $0 \leqslant x \leqslant 87$.

(b) Find the note which has frequency closest to 1400 Hz .

5 A different musical scale has 10 notes.

$$
\begin{array}{llllllllll}
\mathrm{Q} & \mathrm{R} & \mathrm{~S} & \mathrm{~T} & \mathrm{U} & \mathrm{~V} & \mathrm{~W} & \mathrm{X} & \mathrm{Y} & \mathrm{Z}
\end{array}
$$

The frequency of each note is modelled by the function

$$
\mathrm{h}(n)=a \times 2^{b n} \quad \text { where } n \text { is an integer from } 0 \text { to } 29 .
$$

When $n=0$ the note is $\mathrm{Q}_{0}$ and the frequency of this note is 600 Hz . The frequency of the note $\mathrm{Q}_{1}$ is 1200 Hz .
(a) Write down the value of $a$.
(b) Find the value of $b$.
(c) Show that $k \mathrm{~h}(n)=\mathrm{h}(n+1) \quad$ where $k$ is a constant to be found.

6 In musical scales the frequency of the note $\mathrm{P}_{1}$ is two times the frequency of the note $\mathrm{P}_{0}$.
(a) A musical scale has 23 notes.

The frequency of the first note is 75 Hz .
Work out the frequency of the second note.
(b) The first note in another musical scale has a frequency of 100 Hz . The second note has a frequency of 108 Hz .

Find the number of notes in this scale.

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